Lecture 13

In this lecture we begin our introductory study of Logic.

Intro. to Logic

Logic involves statements. Usually they're called propositions.

Eg., let P be the proposition, "All dogs are black."

Let Q be the proposition, "Rover is a dog."

Let R be the proposition, "Rover is black."

We combine propositions using connectives such as AND, OR, IMPLIES, IS EQUIVALENT TO.

We also negate propositions using NOT, which is regarded as a "unary" connective. (The others are binary.)

P and Q: All dogs are black, and Rover is a dog.

P or R: All dogs are black, or Rover is black.

Note

In maths & in logic, "or" means "inclusive or" unless otherwise specified.

Exclusive-or: Today is Monday or Tuesday.
(It can't be both.)

Inclusive-or: You'll pass if you get $\geq 50\%$ this year or next year.

You'll know the answer if I tell you or if someone else tells you. (If we both tell you you still know the answer.)

$P \Rightarrow Q$ —— notation for <u>P implies Q</u>.

(Some texts use $P \rightarrow Q$.)

this means <u>if</u> P is true <u>then</u> Q is true.

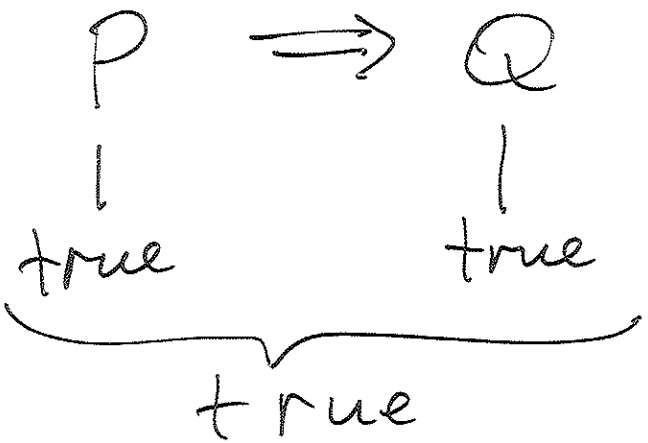
E.g., if a number is bigger than 3 then it is bigger than 2. (True)

$$x > 3 \Rightarrow x > 2$$

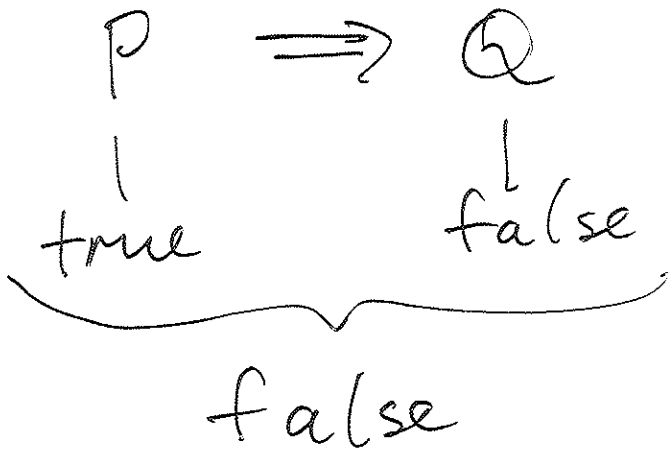
E.g., if 3 is a negative number then the moon is made of green cheese. (True)

This is true because 3 is <u>not</u> a negative no.

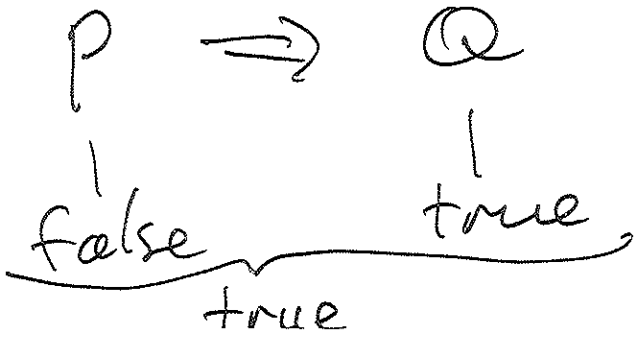
Case I



Case II



Case III



Case IV

$P \Rightarrow Q$

false false

true

So $P \Rightarrow Q$ only fails to be true when P is true and Q is false.

E.g. If today is Monday, tomorrow is Tuesday.

T T

$\therefore$ T

Eg. If today is Monday, tomorrow is Friday.

T F

∴ F

Eg. If this is winter, I'm a monkey's uncle.

F don't know

∴ T

So a false statement implies every other statement.

Also, a true statement is implied by every other statement.

E.g. John Howard is fabulous $\Rightarrow$ today is Monday.

don't know — T

$\therefore$ T

Implications are expressed in many different ways.

P implies Q

$P \Rightarrow Q$

if P then Q

if P, Q

Q if P (Today is Tuesday if yesterday was Sunday.)

P only if Q

P is sufficient for Q

Q is necessary for P

Q (is true) whenever P (is true)

E.g. Being a dog is sufficient for being an animal.

(He is a dog $\Rightarrow$ he is an animal.)

Being an animal is necessary for being a cat.

(You can't be a cat unless you're an animal.)

It is a cat $\Rightarrow$ it is an animal.